

JOHNSON GRANT
IN-61-CR
130535

P-24



**Center for Aeronautical Research
Bureau of Engineering Research
The University of Texas at Austin
Austin, Texas**

CAR 90-3

**APPLICATION OF LANCZOS VECTORS TO CONTROL
DESIGN OF FLEXIBLE STRUCTURES**

by

**Roy R. Craig, Jr.
Tzu-Jeng Su**

**NASA Contract N^o. NAG9-357
December. 1990**

(NASA-CR-191230) APPLICATION OF
LANCZOS VECTORS TO CONTROL DESIGN
OF FLEXIBLE STRUCTURES Interim
Report (Texas Univ.) 24 p

N93-13296

Unclass

G3/61 0130535

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Report No. CAR 90-3
INTERIM REPORT
for
NASA Grant NAG 9-357
NASA - Johnson Space Center
Houston, TX

Center for Aeromechanics Research
(formerly Center for Aeronautical Research)
Bureau of Engineering Research
College of Engineering
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December, 1990

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ACKNOWLEDGEMENT

This work was supported by NASA grant NAG9-357 with the Lyndon B. Johnson Space Center, Houston, Texas. Mr. John Sunkel's interest in this research is greatly appreciated.

ABSTRACT

This report covers research conducted during the first year of the two-year grant. The research, entitled "Application of Lanczos Vectors to Control Design of Flexible Structures" concerns various ways to obtain reduced-order mathematical models for use in dynamic response analyses and in control design studies. This report summarizes research described in the following reports and papers that were written under this contract.

- Su, Tzu-Jeng, *A Decentralized Linear Quadratic Control Design Method for Flexible Structures*, Ref. [5]
- Su, Tzu-Jeng, and Craig, Roy R. Jr., "Controller Reduction by Preserving Impulse Response Energy," Ref. [7]
- Su, Tzu-Jeng, and Craig, Roy R. Jr., "Substructural Controller Synthesis," Ref. [8]
- Su, Tzu-Jeng, and Craig, Roy R. Jr., "Substructuring Decomposition and Controller Synthesis," Ref. [9]
- Craig, Roy R. Jr., and Su, Tzu-Jeng, "A Review of Model Reduction Methods for Structural Control Design," Ref. [10]
- Su, Tzu-Jeng, and Craig, Roy R. Jr., *A Decentralized Linear Quadratic Control Design Method for Flexible Structures*, Ref. [11]
- Craig, Roy R. Jr., "Recent Literature on Structural Modeling, Identification, and Analysis," Ref. [12]

TABLE OF CONTENTS

ACKNOWLEDGEMENT	ii
ABSTRACT	iii
TABLE OF CONTENTS	iv
1. INTRODUCTION	1
2. ABSTRACTS OF TECHNICAL PAPERS	4
2.1 Controller Reduction by Preserving Impulse Response Energy (Ref. 7)	4
2.2 Substructuring Decomposition and Controller Synthesis (Refs. 8,9)	7
2.3 A Review of Model Reduction Methods for Structural Control Design (Ref. 10)	10
2.4 Recent Literature on Structural Modeling, Identification, and Analysis (Ref. 12)	13
3. PROJECTED FUTURE WORK	15
BIBLIOGRAPHY	17

Chapter 1

INTRODUCTION

This report summarizes research accomplished during the first year of a two-year grant on the topic of the application of Krylov and Lanczos vectors to the control of flexible structures. Under a previous contract with NASA-JSC, entitled “Application of Attachment Modes in the Control of Large Space Structures,” [1] Lanczos vectors, and the closely-related Krylov vectors, were first applied to problems related to the control of flexible structures. Results of this research were published in Refs. [2–4], where the use of Krylov vectors as basis vectors and the concept of parameter-matching were combined together to develop Krylov model-reduction algorithms. Algorithms for generating Krylov vectors for general linear systems and for undamped and damped structural systems are presented in these references.

The design of a controller based on a reduced-order system model can lead to three types of control energy spillover: *control spillover*, *observation spillover*, and *dynamic spillover*. The combined effect of the three types of spillover usually degrades the performance of the controller, when it is applied to the full order system, and may even destabilize the closed-loop system. In Refs. [2–4] it is shown that, if model reduction and controller design are based on a Krylov model, then the control and observation spillover terms can be eliminated while leaving only the dynamic spillover to be considered.

The goals stated for research under the present grant are:

1. To develop a theory of reduced-order modeling of general linear systems based on the use of Lanczos vectors, and to apply the theory to the modeling of flexible structures.
2. To address numerical issues that arise in the application of Lanczos vectors to reduced-order modeling, e.g., sensitivity to choice of starting vectors, loss of orthogonality, etc.
3. To develop control system design techniques employing Lanczos modeling of the controlled and residual systems, considering relevant issues such as stability of the closed-loop system, spillover, robustness, and computational requirements.
4. To apply Lanczos-based control system design to general typical problems, e.g., optimal co-located velocity feedback, dynamic output feedback, optimal control of finite-time slewing of a beam, etc.

Much of the research on topics listed under the first two goals above was actually conducted just prior to the start of the present grant and is summarized in Ref. [1]. A theory of reduced-order modeling of general linear systems and of damped and undamped structures (Goal 1) was developed in Ref. [2–5]. Steps that may be taken to address several numerical issues that arise in the application of Lanczos vectors to reduced-order modeling (Goal 2) are described in Ref. [6].

Research that addresses Goals 3 and 4, the development of control system design techniques employing Lanczos vectors, and the application of Lanczos-based control system design, is described in Refs. [3,5,7–11]. Three major topics were studied: the development of a controller reduction method based on Krylov vectors [7], substructuring decomposition and controller synthesis [8,9], and a comparison of Krylov-based model reduction with other model-reduction techniques [10]. This research is fully described in Refs. [5,11], and abstracts of the major published papers are included in Chapter 2 of this report.

The preparation of a survey chapter entitled “Recent Literature on Structural Modeling, Identification and Analysis” [12] was supported, in part, by the present grant.

Finally, Chapter 3 outlines several topics to be studied during the second year of this project.

Chapter 2

ABSTRACTS OF TECHNICAL PAPERS

2.1 Controller Reduction by Preserving Impulse Response Energy (Ref. 7)

This paper presents a controller reduction method that is based on a projection subspace called a *Krylov subspace*. The Krylov subspace is generated by a Krylov recurrence procedure. The reduced-order controller is called an *Equivalent Impulse Response Energy Controller* (EIREC) because it has the same impulse response energy as the full-order controller.

The plant to be controlled is described by the standard first-order state-space form

$$\begin{aligned}\dot{z} &= Az + Bu + N\varpi \\ y &= Cz + v\end{aligned}\tag{2.1}$$

The full-order controller to be reduced is described by

$$\begin{aligned}\dot{q} &= Eq + Fy \\ u &= Gq\end{aligned}\tag{2.2}$$

The impulse response energy is defined as the L^2 energy norm of the impulse response of the controller

$$\mathcal{E} = \|H\|_{L^2}^2 = \text{tr}\left[\int_0^\infty H^T H dt\right]\tag{2.3}$$

where $H = Ge^{Et}F$ is the impulse response of the controller. It is shown that the impulse response energy can further be expressed as

$$\mathcal{E} = \text{tr}\left[\int_0^\infty F^T e^{E^T t} G^T G e^{Et} F dt\right] = \text{tr}[F^T W_o F]\tag{2.4}$$

Or, equivalently,

$$\mathcal{E} = \text{tr} \left[\int_0^\infty G e^{Et} F F^T e^{E^T t} G^T dt \right] = \text{tr} [G W_c G^T] \quad (2.5)$$

where W_o and W_c are the observability and controllability grammians.

Two algorithms for generating projection subspaces for controller system transformation are presented. One algorithm generates a subspace which normalizes the controllability grammian, and the other normalizes the observability grammian. The controller is transformed to new coordinates called *normalized grammian coordinates* by using the projection subspaces. Controller reduction is based upon the representation in the new coordinates. The subspace-generating algorithm is a recursive process with either the F matrix or the G^T matrix as the starting block of vectors. The recursive process is a Krylov recurrence procedure which can be summarized by

$$\begin{aligned} & \text{Choose } L_1 \\ & L_{i+1} = M L_i \end{aligned} \quad (2.6)$$

The first algorithm uses $M = E^{-1}$ and $L_1 = F$ and normalizes the vectors L_i with respect to the observability grammian W_o . The projection subspace is formed as $L = [L_1 \ L_2 \ \dots]$, which satisfies $L^T W_o L = I$. The L subspace is then partitioned into two subspaces $L = [L_R \ L_T]$ with subscripts R and T denoting the retained portion and the truncated portions, respectively. The reduced-order controller is described by

$$\begin{aligned} \dot{\bar{q}}_R &= \bar{E}_R \bar{q}_R + \bar{F}_R y \\ u &= \bar{G}_R \bar{q}_R \end{aligned} \quad (2.7)$$

where $q = L_R \bar{q}$, and where the reduced controller system matrices are given by $\bar{E}_R = L_R^T W_o E L_R$, $\bar{F}_R = L_R^T W_o F$, and $\bar{G}_R = G L_R$. The \bar{F}_R matrix has nonzero elements only in the first block.

The second algorithm uses $M = E^{-T}$ and $L_1 = G^T$ and normalizes the vectors L_i with respect to the controllability grammian. The reduced-order controller system matrices are given by $\bar{E}_R = L_R^T E W_c L_R$, $\bar{F}_R = L_R^T F$, and $\bar{G}_R = G W_c L_R$. The \bar{G}_R matrix has nonzero elements only in the first block.

Both algorithms are useful for constructing reduced-order controllers. The preference depends on the number of actuators and the number of sensors. The algorithms proposed and the reduced-order controllers obtained have the following useful properties.

Property 1: The subspace L is both controllable and observable. If the algorithm terminates before n vectors are generated, then the full-order controller is not minimal. A minimal optimal controller can be produced by projecting the full-order controller onto the L subspace.

Property 2: The reduced-order controller is asymptotically stable if the full-order controller is controllable or observable.

Property 3: If the reduced-order controller is asymptotically stable, then it has the same impulse response energy as the full-order controller.

Property 4: The reduced-order controller matches a set of system parameters. This set of parameters includes the so called low-frequency moments and low-frequency power moments. The low-frequency moments are defined as $GE^{-i}F$, $i = 1, 2, \dots$. The low-frequency power moments are defined as $F^T(E^T)^{-i}W_oE^{-j}F$, and $GE^{-i}W_c(E^T)^{-j}G^T$, $i, j = 1, 2, \dots$

Two examples drawn from other controller reduction literature are used to test the proposed controller reduction method. The Equivalent Impulse Response Reduction Algorithm can produce closed-loop system designs with good

performance. Computationally, the proposed method is economical compared with the other controller reduction methods.

2.2 Substructuring Decomposition and Controller Synthesis (Refs. 8,9)

This paper presents a decentralized control design process called *Sub-structural Controller Synthesis* (SCS). A natural decomposition called substructuring decomposition is used to decompose a flexible structure into several substructures. Then, the linear quadratic optimal control design is carried out to design a subcontroller for each substructure. The final controller for the assembled structure is a global controller, which is assembled from the subcontrollers by using the same assembling scheme as that employed for structure matrices.

Although the method can be applied to structures with more than two substructures, for simplicity a two-component structure is used to demonstrate the formulation. The equations of motion of the two substructures are represented by

$$\begin{aligned} M_i \ddot{x}_i + D_i \dot{x}_i + K_i x_i &= P_i u_i & i = \alpha, \beta \\ y_i &= V_i x_i + W_i \dot{x}_i \end{aligned} \quad (2.8)$$

The dynamics of the the assembled structure (the structure as a whole) is described by

$$\begin{aligned} M \ddot{x} + D \dot{x} + K x &= P u \\ y &= V x + W \dot{x} \end{aligned} \quad (2.9)$$

It is shown that there exists a coupling matrix T which relates x_α , x_β , to x as follows:

$$\begin{Bmatrix} x_\alpha \\ x_\beta \end{Bmatrix} = T x = \begin{bmatrix} T_\alpha \\ T_\beta \end{bmatrix} x \quad (2.10)$$

and the system matrices of the substructures and assembled structure satisfy

$$\begin{aligned} M &= T^T \begin{bmatrix} M_\alpha & 0 \\ 0 & M_\beta \end{bmatrix} T, \quad D = T^T \begin{bmatrix} D_\alpha & 0 \\ 0 & D_\beta \end{bmatrix} T, \quad K = T^T \begin{bmatrix} K_\alpha & 0 \\ 0 & K_\beta \end{bmatrix} T \\ P &= T^T \begin{bmatrix} P_\alpha & 0 \\ 0 & P_\beta \end{bmatrix}, \quad V = \begin{bmatrix} V_\alpha & 0 \\ 0 & V_\beta \end{bmatrix} T, \quad W = \begin{bmatrix} W_\alpha & 0 \\ 0 & W_\beta \end{bmatrix} T \end{aligned} \quad (2.11)$$

For control design purposes, the equations of motion are rewritten in the following first-order forms:

$$\begin{aligned} \begin{bmatrix} D & M \\ M & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} &= \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} P \\ 0 \end{bmatrix} u \\ y &= [V \quad W] \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \end{aligned} \quad (2.12)$$

or

$$\begin{aligned} S\dot{z} &= Az + Bu \\ y &= Cz \end{aligned} \quad (2.13)$$

for the assembled structure. Similarly, the equations of motion of substructures can be represented by

$$\begin{aligned} S_i \dot{z}_i &= A_i z_i + B_i u_i \quad i = \alpha, \beta \\ y_i &= C_i z_i \end{aligned} \quad (2.14)$$

Combination of the two substructure equations above gives the first-order equation of motion of the unassembled structure

$$\begin{aligned} \tilde{S}\dot{\tilde{z}} &= \tilde{A}\tilde{z} + \tilde{B}u \\ y &= \tilde{C}\tilde{z} \end{aligned} \quad (2.15)$$

with the system matrices in the following block diagonal form

$$\tilde{S} = \begin{bmatrix} S_\alpha & 0 \\ 0 & S_\beta \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A_\alpha & 0 \\ 0 & A_\beta \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_\alpha & 0 \\ 0 & B_\beta \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C_\alpha & 0 \\ 0 & C_\beta \end{bmatrix}$$

and

$$\tilde{z} = \begin{Bmatrix} z_\alpha \\ z_\beta \end{Bmatrix}$$

It can be shown that the system matrices of the unassembled structure and the assembled structure satisfy

$$S = \tilde{T}^T \tilde{S} \tilde{T} \quad A = \tilde{T}^T \tilde{A} \tilde{T} \quad B = \tilde{T}^T \tilde{B} \quad C = \tilde{C} \tilde{T} \quad (2.16)$$

with

$$\tilde{T} = \begin{bmatrix} T_\alpha & 0 \\ 0 & T_\alpha \\ T_\beta & 0 \\ 0 & T_\beta \end{bmatrix}$$

The control design method is the LQG theory with the performance indices

$$J_i = \lim_{t \rightarrow \infty} \frac{1}{2} E[\dot{x}_i^T M_i \dot{x}_i + x_i^T M_i x_i + u_i^T R_i u_i] \quad i = \alpha, \beta \quad (2.17)$$

for substructures being minimized. The subcontroller is in the form

$$\begin{aligned} S_i \dot{q}_i &= (A_i + B_i G_i^o - F_i^o C_i) q_i + F_i^o y_i & i = \alpha, \beta \\ u_i &= G_i^o q_i \end{aligned} \quad (2.18)$$

with F_i^o and G_i^o the optimal control gain and optimal filter gain matrices. In a more compact form, the controller equation for the unassembled system is written as

$$\begin{aligned} \tilde{S} \dot{\tilde{q}} &= (\tilde{A} + \tilde{B} \tilde{G}^o - \tilde{F}^o \tilde{C}) \tilde{q} + \tilde{F}^o y \\ u &= \tilde{G}^o \tilde{q} \end{aligned} \quad (2.19)$$

with

$$\tilde{G}^o = \begin{bmatrix} G_\alpha^o & 0 \\ 0 & G_\beta^o \end{bmatrix}, \quad \tilde{F}^o = \begin{bmatrix} F_\alpha^o & 0 \\ 0 & F_\beta^o \end{bmatrix} \quad (2.20)$$

The unassembled closed-loop system is an optimal control system since each subcontroller is optimal for its associated substructure.

The last step is to assemble the subcontrollers by using the same coupling scheme as that used for assembling the substructures. The assembled controller

for the assembled structure is represented by

$$\begin{aligned} S\dot{q} &= (A + BG^* - F^*C)q + F^*y \\ u &= G^*q \end{aligned} \quad (2.21)$$

with

$$F^* = \tilde{T}^T \tilde{F}^o, \quad G^* = \tilde{G}^o \tilde{T} \quad (2.22)$$

where superscript $*$ denotes that the controller is not optimal but is considered as suboptimal.

One advantage of using Substructural Controller Synthesis to design a controller is that an SCS controller is highly adaptable. For a structure with varying configuration or varying mass and stiffness properties, like some space structures, the Substructural Controller Synthesis method may be especially efficient. The SCS controller can be updated economically by simply carrying out redesign of subcontrollers associated with those substructures that have changed. On the other hand, for a controller based on a centralized design scheme, a slight change of the structure may require a full-scale controller redesign.

An LQGSCS Algorithm is presented in the paper to summarize the Substructural Controller Synthesis procedure. Also a two-component planar truss structure with non-colocated actuators and sensors is used to demonstrate the applicability of the proposed method.

2.3 A Review of Model Reduction Methods for Structural Control Design (Ref. 10)

In this paper, several frequently used model reduction methods are briefly reviewed. The methods reviewed include: modal truncation, balanced model

reduction, balanced gain approach, Krylov model reduction, and Ritz vectors and mixed-mode method.

Among a myriad of existing model reduction methods for structural dynamics systems, modal truncation may be the most popular approach. A modal representation has many advantages: modal frequencies represent resonances of the structure, equations of motion are uncoupled implying a saving of computation time, and modal data can be identified and validated by vibration test. However, selection of modes to be retained in the reduced model may not be an easy task. The simplest approach would be to include all modes within the frequency range of interest. For a large space structure with closely-spaced frequencies, this simplest approach may produce a reduced model whose size is still too large to handle.

An efficient modal truncation criterion is based on balanced singular values. If the structure frequencies are sufficiently separated and modal damping is very small, then modal representation of a structural dynamics system is approximately balanced. The approximate balanced singular values are calculated by

$$\sigma_i \approx \frac{\sqrt{p_i p_i^T (v_i^T v_i + \lambda_i^2 w_i^T w_i)}}{4\zeta_i \lambda_i^2} \quad (2.23)$$

where p_i is the i -th row of force distribution matrix, v_i and w_i are the i -th columns of the displacement and velocity sensor distribution matrices, and λ_i and η_i are the i -th modal frequency and damping ratio. Balanced model reduction is performed on modal coordinates.

Other than balanced singular values, balanced gains also can serve as a

basis for modal truncation. Balanced gains are defined as

$$g_i = \frac{\sqrt{p_i p_i^T (v_i^T v_i + \lambda_i^2 w_i^T w_i)}}{\lambda_i} \quad (2.24)$$

The balanced gain approach is an optimal modal truncation method in the L^2 sense. It is closely related to the modal cost analysis.

In addition to normal modes, there are other Ritz vector superposition methods for dynamic analysis of structures. In Ref. [4], Su and Craig presented a Ritz vector method called Krylov model reduction. Krylov vectors are system static modes generated by a recurrence procedure. For undamped structural dynamics systems, the Krylov procedure is

$$Q_{j+1} = K^{-1} M Q_j \quad (2.25)$$

For damped systems, the Krylov procedure is

$$\begin{Bmatrix} Q_{j+1}^d \\ Q_{j+1}^v \end{Bmatrix} = \begin{bmatrix} -K^{-1}D & -K^{-1}M \\ I & 0 \end{bmatrix} \begin{Bmatrix} Q_j^d \\ Q^v \end{Bmatrix} \quad (2.26)$$

Krylov reduced models match a set of system parameters called low-frequency moments.

Other than using only normal modes or only Ritz vectors, a mixed-mode method combines some dominant normal modes and some static modes in the basis for model reduction. Static modes are a system's deflection shapes associated with imposed force distribution vectors. Krylov vectors can be considered to be system static modes. Recently, several numerical experiments have shown that by augmenting a modal basis with some Ritz vectors, fidelity of the reduced model can be substantially improved [13].

In this paper, a plane truss structure with closely-spaced frequencies and light modal damping is used to compare different reduced-order models. Open-loop comparison includes the L^2 error norm of the impulse response function and approximation of the output frequency response function. The control design comparison includes closed-loop stability and control performance. For the example studied, the balanced gain approach produces reduced-order models that approximate the impulse response better than any other truncation criterion. However, for closed-loop comparison, modal truncation by preserving the lowest frequency modes yields more stable closed-loop designs than other methods.

2.4 Recent Literature on Structural Modeling, Identification, and Analysis (Ref. 12)

This paper, which surveys literature from 1980 to the present related to the topics of modeling, identification, and analysis of large space structures, provides a list of over 240 references. The topic of “Mathematical Modeling of Large Space Structures” includes sections dealing with the following subjects:

- Continuum Models
- Model Order Reduction
- Substructuring; Component Synthesis
- Computational Techniques
- Waves vs Modes

- Localization: Local vs Global Behavior
- Nonlinearity: Joints
- Damping
- Modeling Errors and Parameter Uncertainty

Under the topic “System Identification; Model Verification and Model Updating” the following subjects are surveyed:

- Experimental Modal Analysis
- Model Verification: Model Error Localization and Model Updating
- Scale Modeling of Large Space Structures
- Damage Detection
- On-Orbit System Identification

Chapter 3

PROJECTED FUTURE WORK

During the second year of this project, the research will continue to focus on investigation of important issues in structure/control interaction problems, and will specifically focus on the following topics:

1. To address stability issues in substructure-based control design, and to develop an augmented global controller in order to reduce the influence of the substructures on each other in the closed-loop system, such that stability and performance of the substructure-based controllers is enhanced.
2. To develop a reduced-order observer in second-order form, which is to be used as the feedback basis in the control design for large scale structural dynamics systems.
3. To incorporate the Krylov model reduction and component mode synthesis method with the substructural control design approach, and to illustrate the feasibility and efficiency of the Substructural Controller Synthesis method when it is used to design controllers for practical large flexible space structures.
4. To apply the Krylov model reduction and the substructural controller synthesis method to the controller design for multi-flexible-bodies and

articulated mechanical systems, which involve reduced-order modeling, precision-pointing, and vibration suppression.

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